50X1-HUM

Page Denied

Next 1 Page(s) In Document Denied

COHERENT TRANSITION RADIATION OF CURRENTS AND CHARGES

V. N. Tsytovich

Translated from Zhurnal Tekhnicheskoi Fiziki, Vol. 31, No. 8, pp. 923-935, August, 1961
Original article submitted October 19, 1960

It is shown that the coherent transition radiation of currents and charges can be responsible for a large part of the electromagnetic mass if $\lambda_0 = c/\omega_0$ (ω_0 is the Langmuir frequency of the medium) becomes comparable with the transverse dimensions of the bunch. The dependence of radiation intensity on the longitudinal dimensions of the bunch is also considered.

It has been shown by Ginzburg and Frank [1] that the passage of a charge through a boundary which separates two media will, in general, give rise to so-called transition radiation; in the ultrarelativistic case ($\gamma = \frac{1}{\sqrt{1-\beta^2}} \gg 1$) this radiation is directed primarily along the direction of motion of the charge. If multiple scattering is neglected [2, 3] the intensity of the transition radiation is proportional to the energy of the charge [3, 4].

Below, we consider the radiation produced by a charge bunch; if the conditions for coherence are satisfied this radiation will differ from the sum of the radiation due to the individual charges. Inasmuch as the radiation is primarily in the forward direction in the ultrarelativistic limit, only the longitudinal dimensions of the bunch enter into the coherence condition in this case. Up to some critical dimension l_c the intensity increases in proportion to the squares of the number of particles in the bunch; in other words, the radiation reaction force acting on each particle in a bunch with dimensions smaller than l_c increases in proportion to the number of particles.

If the transverse dimensions $\underline{a}$ are comparable with $\lambda_0 = c/\omega_0$, $\omega_0^2 = 4\pi Ne^2/m$, the higher multipole moments of the bunch become of importance in the transition radiation and an expansion in multipole moments can not be used. This analysis is carried out below.

Interest in transition radiation of charge bunches stems particularly from the possibility of using transition radiation as a source for electromagnetic waves [5]; if the coherence condition is satisfied the intensity of the radiation can be increased considerably.

The coherent effects associated with the passage of a charge bunch through a medium were first pointed out by Veksler [6]. B. M. Bolotovskii has considered the coherent Cerenkov radiation for a charge bunch [7]; Morozov, and later Bogdankevich [8], have also considered the Cerenkov radiation due to currents. Up to the present time, however, coherent effects associated with transition through a separation boundary between two media have not been considered.

The analysis carried out by Bolotovskii indicates that the coherent radiation reaction force leads to a worsening of the coherence condition: specifically, to an elongation of the bunch so that the coherence condition is violated after some time interval. This same result has been obtained by Rabinovich and Ioganson [9], who have considered the coherent radiation of bunches in the synchrotron. The physical basis of this effect lies in the fact that because the radiation is directed forward, the following particles experience a reaction force due to the particles which precede them but the converse is not true. As a result, the reaction force acting on an individual particle in the bunch increases along the length, and this leads to stretching and smearing of the bunch.

However, this result is not a universal one. In particular, in transition radiation of bunches the coherence effect can cause compression of the bunches. This follows because the coherent force acting on an individual particle in the bunch (as it passes through the boundary separating two media) does not coincide with the radiation reaction force because part of the work of the force goes into "renormalization" of the electromagnetic mass of the bunch. In other words, the electromagnetic mass (energy of the electromagnetic field) is different in different media with different ε for a uniformly moving bunch. In passing through the separation boundary, in addition to finding radiation reaction forces, one expects to find additional forces which are associated with the renormalization of the mass.

If sufficiently large currents are excited in the bunch, it is possible to arrive at a situation in which compression takes place at both boundaries, i.e., upon entrance to and exit from the medium.

Below, for this reason, we consider the radiation due to charge bunches and due to currents.

1. Transition Radiation of Charge Bunches

The current density for a uniformly moving bunch whose charge distribution is symmetric with respect to the axis along which it moves is

$$j_z = \rho v_z;$$

$$\rho = \frac{Q}{4\pi v}\int_{-\infty}^{+\infty} s_\omega e^{i\frac{\omega}{v}(z-vt)}\,d\omega \int_0^\infty \rho_k J_0(kr)\,dk. \quad (1)$$

where J_0 is a Bessel function of zero order. It is also assumed that the charge density in the system moving with the charge is a product of two factors which depend on z and r separately:

$$\rho_{mov} = s_{mov}(z)\rho_{mov}(r);$$

$$s_\omega = \int_{-\infty}^{\infty} s_{mov}(z) e^{-i\frac{\omega}{v\gamma}z}\,dz,$$

$\rho_k = \int_0^\infty \rho_{mov}(r) J_0(kr) 2\pi kr\ dk.$ If the bunch is cylindrical with radius a and length l, then

$$\rho_k = 2\pi a J_1(ka);\quad s_\omega = \frac{\omega_l}{\omega}\sin\frac{\omega}{\omega_l};\quad \omega_l = \frac{v\gamma}{l}$$

where Q is the charge which passes through a unit area of the cross section. If the bunch has a cross section in the form of a thin ring, then $\rho_k = 2\pi ak\, J_0(ak)\, g(k)$, where $g(k)$ is a form factor which cuts off higher k of order $1/r$ (r is the difference between the inner and outer radii of the ring), Q is the charge per unit length around the circle of radius a. Using Eq. (1), it is easy to write the potentials for the usual Lorentz gauge

$$A_z = \varepsilon\beta\varphi;$$

$$\varphi = \frac{Q}{\pi v}\int \frac{1}{\left(\frac{\omega^2}{v^2} - \lambda_\varepsilon^2\right)\varepsilon} e^{i\frac{\omega}{v}(z-vt)} s_\omega \rho_k J_0(kr)\,dk\,d\omega, \quad (2)$$

where $\lambda_\varepsilon^2 = \frac{\omega^2}{c^2}\varepsilon - k^2$.

We now consider the transition from vacuum to medium. In the vacuum, as in the medium, the inhomogeneous solution of the potential equations will be of the

where in the vacuum, ...

$$\varphi_{fr.} = \frac{Q}{\pi}\int \frac{1}{\varepsilon\omega}\Phi_\varepsilon e^{i(\lambda_\varepsilon z-\omega t)} s_\omega\rho_k J_0(kr)\,dk\,d\omega, \quad (3)$$

$$A_{fr.} = \frac{Q}{\pi c}\int \Phi_\varepsilon e^{i(\lambda_\varepsilon z-\omega t)} s_\omega \rho_k J_0(kr)\,dk\,d\omega, \quad (4)$$

Here, $\mathrm{Im}\lambda_\varepsilon > 0$ (the medium is located in the region $z \geq 0$). Similarly, we can write the free field in the vacuum replacing Φ_ε by Φ and λ_ε by λ with the condition $\mathrm{Im}\lambda < 0$. The boundary conditions at $z = 0$ allow us to find Φ and Φ_ε

$$\Phi = \frac{1}{\lambda_\varepsilon - \varepsilon\lambda}\left\{\frac{\varepsilon\frac{\omega}{v} - \lambda_\varepsilon}{\frac{\omega^2}{v^2} - \lambda^2} - \frac{1}{\frac{\omega}{v} + \lambda_\varepsilon}\right\}, \quad (5)$$

$$\Phi_\varepsilon = \frac{1}{\varepsilon\lambda - \lambda_\varepsilon}\left\{\frac{\frac{\omega}{v} - \varepsilon\lambda}{\frac{\omega^2}{v^2} - \lambda_\varepsilon^2} - \frac{\varepsilon}{\frac{\omega}{v} + \lambda}\right\}. \quad (6)$$

Because of symmetry, the only non-vanishing force component is

$$F_z = \int \rho E_z 2\pi r\,dr\,dz$$

$$= \frac{Q}{2\pi v}\int s_\omega \rho_k E_z e^{i\frac{\omega}{v}(z-vt)} J_0(kr)\,dk\,d\omega\,r\,dr\,dz. \quad (7)$$

The work of the free field corresponds to losses due to Cerenkov radiation and the radiation of longitudinal waves. The transition effect is then connected with the work of the free field, and in the medium

$$E_{fr.,z} = \frac{iQ}{\pi}\int \frac{k'^2}{\varepsilon'\omega'}\Phi'_\varepsilon e^{i(\lambda'_\varepsilon z - \omega' t)} s_{\omega'}\rho_{k'} J_0(k'r)\,dk'd\omega'. \quad (8)$$

Substituting Eq. (8) in Eq. (7) and integrating with respect to time and r, we obtain the δ-functions $\delta(\omega + \omega')$ and $\delta(k - k')$ which remove the integration with respect to ω' and k'

$$W_z = \int_{-\infty}^{\infty} F_z v\,dt$$

$$= \frac{iQ^2}{\pi}\int \frac{k}{\varepsilon\omega}\Phi_\varepsilon |s_\omega|^2 \rho_k^2\,d\omega\,dk\,dz\,e^{i\left(\lambda_\varepsilon - \frac{\omega}{v}\right)z}. \quad (9)$$

Here we have made use of the obvious relation $s_{-\omega} = s_\omega^*$. The integration over z in Eq. (9) must be carried out from 0 to ∞. Taking $\mathrm{Im}\lambda_\varepsilon > 0$, we have

$$W_z = -\frac{Q^2}{\pi}\int \frac{k}{\varepsilon\omega}\Phi_\varepsilon \frac{1}{\lambda_\varepsilon - \frac{\omega}{v}} \ldots$$

$$W_s = \frac{Q^2}{\pi}\int \frac{k}{\omega}\Phi \frac{1}{\lambda - \frac{\omega}{v}} |s_\omega|^2 \rho_k^2 d\omega dk. \qquad (11)$$

The form factor $|s_\omega|^2$ allows us to investigate the dependence of this work on the longitudinal dimensions of the bunch whereas ρ_k^2 allows us to study the effect of the dimensions. We consider the ultrarelativistic limit $\gamma = \frac{1}{\sqrt{1-\beta^2}} \gg 1$. In this case high frequencies are excited and we may assume $\epsilon - 1 \simeq \frac{\omega_0^2}{\omega^2} \ll 1$; $\omega_0^2 = \frac{4\pi N e^2}{m}$. Furthermore, it is evident that we can take $\omega \gg ck$ because the radiation is primarily in the forward direction. Then, as an approximation we have $\lambda \simeq -\frac{\omega}{c}$; $\lambda_s \simeq \frac{\omega}{c}$, except for the denominators, which vanish if this holds,

$$\Phi_s \simeq \frac{1}{\frac{\omega^2}{v^2} - \lambda^2} - \frac{1}{\frac{\omega^2}{v^2} - \lambda_s^2};$$

$$\frac{1}{\left(\lambda_s - \frac{\omega}{v}\right)\epsilon\frac{\omega}{c}} \simeq \frac{2}{\lambda_s^2 - \frac{\omega^2}{v^2}}. \qquad (12)$$

In the ultrarelativistic limit the work in vacuum is negligibly small compared with the work in the medium, so that the total work is given by

$$W \simeq W_s \simeq -\frac{2Q^2}{\pi c}\int k\rho_k^2 |s_\omega|^2 \left\{ \frac{1}{\left(\lambda_s^2 - \frac{\omega^2}{v^2}\right)^2} - \frac{1}{\left(\lambda_s^2 - \frac{\omega^2}{v^2}\right)\left(\lambda^2 - \frac{\omega^2}{v^2}\right)} \right\} d\omega dk. \qquad (13)$$

If the longitudinal dimensions are sufficiently small (cf. below) $|s_\omega|^2 \approx 1$ and integration over ω can be carried out in elementary fashion by taking residues

$$W_{\text{coh.}} = -Q^2\gamma \int_0^\infty k\rho_k^2 \left\{ \frac{1}{\left(k^2 + \frac{\omega_0^2}{c^2}\right)^{3/2}} + 2\frac{c^2}{\omega_0^2}\left(\frac{1}{\sqrt{k^2 + \frac{\omega_0^2}{c^2}}} - \frac{1}{k} \right) \right\} dk. \qquad (14)$$

In this way we determine the coherent work done by the forces upon entrance into the medium for short bunches with any distribution of charge over radius. In the case of a circular bunch $\rho_k^2 = 4\pi^2 a^2 k^2 J_0^2(ka)\, g^2(k)$. The function $\pi ka J_0^2(ka)$ at large values of ka behaves as $1 + \cos(2ka + \varphi) \approx 1$ [since the rapidly varying part of the integral in Eq. (14) can be neglected]; at small values of ka this function goes as $\pi/4\,(ka)^3 \ll 1$. In other words, the main contribution occurs when $ka \gg 1$, and for making an estimate in (14) we can write $\pi ka J_0^2(ka) = 1$, carrying out the integration over k to $k_{min} \approx 1/a$ and $k_{max} \approx 1/r$ [factor $g^2(k)$].

$$W_{\text{ooh}} \approx -4\pi a Q^2\gamma \int_{k_{min}}^{k_{max}} dk \left\{ \frac{1}{\left(k^2 + \frac{\omega_0^2}{c^2}\right)^{3/2}} + 2\frac{c^2}{\omega_0^2}\left(\frac{1}{\sqrt{k^2 + \frac{\omega_0^2}{c^2}}} - \frac{1}{k} \right) \right\}. \qquad (15)$$

Whence,

$$W_{\text{coh.}} = -4\pi a Q^2\gamma \times \left\{ -\frac{k_{max}}{\sqrt{k_{max}^2 + \frac{\omega_0^2}{c^2}}} + \frac{k_{min}}{\sqrt{k_{min}^2 + \frac{\omega_0^2}{c^2}}} + \frac{c^2}{\omega_0^2} k_{max}\sqrt{k_{max}^2 + \frac{\omega_0^2}{c^2}} - \frac{c^2}{\omega_0^2} k_{min} \times \sqrt{k_{min}^2 + \frac{\omega_0^2}{c^2}} - \frac{c^2}{\omega_0^2} k_{max}^2 + \frac{c^2}{\omega_0^2} k_{min}^2 \right\}. \qquad (16)$$

The optimum value of the work corresponds to

$$W_{\text{coh.}} \simeq 2\pi a Q^2\gamma = \frac{e^2N^2}{2\pi a}\gamma; \quad Q = -\frac{eN}{2\pi a} \qquad (17)$$

where N is the number of particles in a bunch, e is the charge on an individual particle.* In other words, the work is of order $\Delta m\gamma c^2$ where Δm is the electromagnetic mass of the bunch. This same result applies for a cylindrical bunch if $c/\omega_0 \sim a$.

In the other case $c/\omega_0 \gg a$ we can write $J_0(ka) = 1$

$$W = -4\pi^2 a^2 Q^2\gamma \int_0^\infty k^3 dk \left\{ \frac{1}{\left(k^2 + \frac{\omega_0^2}{c^2}\right)^{3/2}} \right.$$

*If charges of opposite sign are present we have $N = N_- - N$

$$+2\frac{c^2}{\omega_0^2}\left(\frac{1}{\sqrt{k^2+\frac{\omega_0^2}{c^2}}}-\frac{1}{k}\right)\Bigg\}=\frac{2}{3}e^2N^2\gamma\frac{\omega_0}{c}. \quad (18)$$

The last result coincides with the results obtained in [3], in which we must replace the particle charge e by the charge of a bunch Ne. The work in (18), corresponding to a point charge, is small compared with (17) and represents a small fraction of the electromagnetic mass multiplied by γ.

Thus, the work due to forces on a charge bunch increases appreciably when the dimensions of the bunch become comparable with $\lambda_0 = c/\omega_0$. This also holds true for transition radiation; to obtain the latter it is necessary to determine the "renormalization" of the mass.

We now find the difference in the energy of the electromagnetic field of the bunch in vacuum and in the medium. In the medium we substitute in

$$\mathcal{E}=\frac{1}{8\pi}\int H^2dv+\frac{1}{4\pi}\int_{-\infty}^{t}dt\int dv\mathbf{E}\frac{\partial\mathbf{D}}{\partial t} \quad (19)$$

the expression for the potentials (2)

$$\mathcal{E}=\frac{Q^2v}{2\pi c^2}\int k\rho_k^2|\varepsilon_\omega|^2\frac{d\omega dk}{\left(\frac{\omega^2}{v^2}-\lambda_\varepsilon^2\right)^2}$$

$$+\frac{Q^2}{2\pi v}\int\frac{\left(1+\frac{\omega_0^2}{\omega^2}\right)\rho_k^2|\varepsilon_\omega|^2}{\varepsilon^2\left(\frac{\omega^2}{v^2}-\lambda_\varepsilon^2\right)^2}\left[k+\frac{\omega^2}{v^2k}(1-\beta^2\varepsilon)\right]d\omega dk. \quad (20)$$

In the ultrarelativistic limit we have

$$\mathcal{E}\simeq\frac{Q^2}{\pi c}\int k\rho_k^2|\varepsilon_\omega|^2\frac{d\omega dk}{\left(\frac{\omega^2}{v^2}-\lambda_\varepsilon^2\right)^2}. \quad (21)$$

In order to renormalize the mass, we must subtract the energy of the field in vacuum ($\lambda_\omega \to \lambda$) from the energy of the field in the medium (21). For the case $|\varepsilon_\omega|^2 \simeq 1$ we easily find

$$\Delta\mathcal{E}=\frac{Q^2\gamma}{2}\int\rho_k^2kdk\left[\frac{1}{\left(k^2+\frac{\omega_0^2}{c^2}\right)^{3/2}}-\frac{1}{k^3}\right]. \quad (22)$$

When $\lambda \ll a$, for the circular case we have

$$\Delta\mathcal{E}=2\pi aQ^2\gamma\int_{k_{\min}}^{k_{\max}}\left[\frac{k^2}{\left(k^2+\frac{\omega_0^2}{c^2}\right)^{3/2}}-\frac{1}{k}\right]dk. \quad (23)$$

For the optimum case $k_{\min} \ll \omega_0/c \ll k_{\max}$

$$\Delta\mathcal{E}=-\frac{e^2N^2}{2\pi a}\gamma\left\{1+\ln\frac{\omega_0}{2ck_{\min}}\right\}. \quad (24)$$

In the other limiting case $\lambda_0 \gg a$, $J_0(ka) \simeq 1$

$$\Delta\mathcal{E}=\frac{e^2N^2}{2}\int_0^\infty\left[\frac{k^3}{\left(k^2+\frac{\omega_0^2}{c^2}\right)^{3/2}}-1\right]dk$$

$$=-e^2N^2\frac{\omega_0}{c}. \quad (25)$$

This result differs from the result in [3] in that the particle charge e is replaced by the charge of the bunch Ne. The intensity of the transition radiation is easily found from the conservation law

$$-\frac{c}{4\pi}\int s_r d\sigma=\frac{dW}{dt}+\frac{d\mathcal{E}}{dt}. \quad (26)$$

We have

$$I\equiv\frac{c}{4\pi}\int_{-\infty}^{\infty}dt\int s_r d\sigma=-\Delta\mathcal{E}-W. \quad (27)$$

In the case of interest, it follows from Eqs. (17) and (24) that

$$I=\frac{e^2N^2}{2\pi a}\gamma\ln\frac{a}{2\lambda_0}, \quad (28)$$

where $\lambda_0 = c/\omega_0$. It should be noted that the last expression differs from the original electromagnetic mass of the bunch multiplied by γc^2 only in the value under the logarithm [in particular, in Eq. (28) we have $a/2\lambda_0$ rather than a/r]. If the medium is a thick plate the work is conserved at the other boundary when the bunch leaves the medium. We can find this quantity if we assume that the renormalization of the mass must appear with opposite sign

$$W'=\Delta\mathcal{E}-I=-\frac{e^2N^2}{2\pi a}\gamma\left(1+\ln\frac{a}{2\lambda_0}\right). \quad (29)$$

For a thick plate, the total work at both boundaries is equal to twice the intensity of the transition radiation (28) and is greater than the electromagnetic mass of the bunch multiplied by γc^2, i.e., the bunch, whose greatest mass is electromagnetic, must be strongly retarded by the radiation.†

2. Transition Radiation from Currents

In a bunch which contains charges of different sign the existence of a current may provide stability of the bunch and thereby increase the coherent effect; that is to say, a larger number of charges can be concentrated

† Here we are speaking roughly since the given motion is approximate to begin with.

within the dimensions l_k. We shall consider only circular currents directed perpendicularly to the motion of the bunch

$$j_\varphi = \frac{J}{4\pi^2 v}\int_{-\infty}^{\infty} s_\omega e^{i\frac{\omega}{v}(z-vt)}\, d\omega \int_0^{\infty} j_k J_1(kr)\, dk \quad (30)$$

where s_ω characterizes the current distribution over length while j_k is the distribution over radius

$s_\omega = \int_{-\infty}^{\infty} j_{mov}(z) e^{-i\frac{\omega}{v\gamma}z} dz$. In the moving system

$j_{mov} = j_{mov}(z)\, j_{mov}(r)$; $j_k = \int_0^{\infty} 2\pi r k j_{mov}(r) J_1(kr)\, dr$.

For a cylindrical bunch of length $2l$, the quantity s_ω is the form

$$s_\omega = \frac{\omega_l}{\omega}\sin\frac{\omega}{\omega_l}; \quad \omega_l = \frac{v\gamma}{l}.$$

If the transverse cross section is a thin ring of radius a, then $j_k = 2\pi k J_1(ak)\, g(k)$ where J is the total current and J_1 is a Bessel function of the first kind.

The radiation due to the current is made up of the contributions due to the radiation of the charge. The self field and the free field are of the form

$$A_\varphi = \frac{J}{c\pi v}\int \frac{s_\omega j_k}{\frac{\omega^2}{v^2} - \lambda_\varepsilon^2} e^{i\frac{\omega}{v}(z-vt)} J_1(kr)\, dk\, d\omega, \quad (31)$$

$$A_\varphi^{fr.} = \frac{J}{c\pi v}\int s_\omega j_k J_1(kr)\, \Phi_\varepsilon e^{i\lambda_\varepsilon z - i\omega t} dk\, d\omega, \quad (32)$$

Similar considerations hold for the field in vacuum ($\lambda_\varepsilon \to \lambda$; $\Phi_\varepsilon \to \Phi$).

From the boundary conditions we have

$$\Phi_\varepsilon = \frac{\lambda + \lambda_\varepsilon}{\left(\lambda_\varepsilon^2 - \frac{\omega^2}{v^2}\right)\left(\lambda + \frac{\omega}{v}\right)};$$

$$\Phi = \frac{\lambda + \lambda_\varepsilon}{\left(\lambda^2 - \frac{\omega^2}{v^2}\right)\left(\lambda_\varepsilon + \frac{\omega}{v}\right)}. \quad (33)$$

The self field of the current does no work. The total work of the free field in the plasma and in the vacuum is obtained from (32), (33)

$$W_\varepsilon = \int_{-\infty}^{\infty} dt \int_{z>0} dv\, j_\varphi E_\varphi$$

$$= -\frac{J^2}{c^2}\int \frac{|s_\omega|^2 j_k^2 \omega \Phi_\varepsilon}{v^2 \pi k\left(\lambda_\varepsilon - \frac{\omega}{v}\right)}\, d\omega\, dk, \quad (34)$$

$$W_0 = \int_{-\infty}^{\infty} dt \int_{z<0} dv\, j_\varphi E_\varphi$$

$$= \frac{J^2}{c^2}\int \frac{|s_\omega|^2 j_k^2 \Phi_\omega}{v^2 \pi k\left(\lambda - \frac{\omega}{v}\right)}\, d\omega\, dk. \quad (35)$$

For the coherent case $|s_\omega|^2 \simeq 1$; in the ultrarelativistic limit we have

$$\left.\begin{aligned}
&\Phi_\varepsilon \simeq -\frac{\omega_0^2}{c^2\left(\lambda_\varepsilon^2 - \frac{\omega^2}{v^2}\right)\left(\lambda^2 - \frac{\omega^2}{v^2}\right)};\\
&\frac{1}{\lambda_\varepsilon - \frac{\omega}{v}} \simeq 2\frac{\omega}{c}\frac{1}{\left(\lambda_\varepsilon^2 - \frac{\omega^2}{v^2}\right)},\\
&W = W_0 + W_\varepsilon\\
&\simeq \frac{2J^2\omega_0^2}{\pi c^2 c^2}\int \frac{j_k^2}{ck}\frac{\frac{\omega^2}{c^2}}{\left(\lambda_\varepsilon^2 - \frac{\omega^2}{v^2}\right)^2\left(\lambda^2 - \frac{\omega^2}{v^2}\right)}\, d\omega\, dk.
\end{aligned}\right\} \quad (36)$$

The integration over frequency is elementary

$$W \simeq \frac{J^2\gamma^3}{c^2}\int \frac{j_k^2}{k}\left\{2\frac{c^2}{\omega_0^2}\left(\sqrt{k^2 + \frac{\omega_0^2}{c^2}} - k\right) - \frac{1}{\sqrt{k^2 + \frac{\omega_0^2}{c^2}}}\right\} dk. \quad (37)$$

For a ring current, if $\lambda_0 \ll a$, then $j_k^2 \simeq 4\pi a k$ and $k_{min} \ll \omega_0/c \ll k_{max}$

$$W \simeq 2\pi a\frac{J^2\gamma^3}{c^2}. \quad (38)$$

In the present case the work is proportional to the cube of the energy of the bunch $\sim \gamma^3$. Since the current strength in the moving system J_{mov} is $J\gamma$, it is apparent that (38) is approximately the electromagnetic mass multiplied by γc^2. In the other limiting case $\lambda_0 \gg a$, we can write $j_k^2 = 4\pi^2 a^2 k^2 J_1^2(ak) \simeq 4\pi^2 a^4 k^4$ and Eq. (37) yields

$$W=\frac{4J^2\pi^2a^4}{c^3}\int_0^\infty k^3\left\{2\frac{c^2}{\omega_0^2}\sqrt{k^2+\frac{\omega_0^2}{c^2}}-2k\frac{c^2}{\omega_0^2}-\frac{1}{\sqrt{k^2+\frac{\omega_0^2}{c^2}}}\right\}dk$$

$$=\mu^2\frac{\omega_0^2}{c^2}\gamma\left(k_{max}-\frac{8}{5}\frac{\omega_0}{c}\right);\qquad \mu=\frac{J\gamma}{c}\pi a^2, \tag{39}$$

that is to say, a result corresponding to a magnetic moment μ. The mass is renormalized by substituting Eq. (31) in Eq. (19):

$$\mathcal{E}=\frac{J^2}{2\pi v c^2}\int |s_\omega|^2\frac{j_k^2}{k}$$

$$\times\frac{\left(\frac{\omega^2}{v^2}+\frac{\omega^2}{c^2}+k^2+\frac{\omega_0^2}{c^2}\right)}{\left(\lambda_s^2-\frac{\omega^2}{v^2}\right)^2}d\omega dk. \tag{40}$$

In the ultrarelativistic limit, for $|s_\omega|^2=1$ we subtract from Eq. (40) the value in vacuum

$$\Delta\mathcal{E}=\frac{J^2\gamma^3}{2c^2}\int\frac{j_k^2}{k}\left(\frac{1}{\sqrt{k^2+\frac{\omega_0^2}{c^2}}}-\frac{1}{k}\right). \tag{41}$$

Or, when $\lambda_0 \ll a$ and $\lambda_0 \gg 1/k_{max}$.

$$\Delta\mathcal{E}=-\frac{J^2\gamma^3}{c^2}\ln\frac{a}{2\lambda_0}. \tag{42}$$

The radiation intensity is found from the conservation law (27)

$$I=2\pi a\frac{J^2\gamma^3}{c^2}\ln\frac{a}{2e\lambda_0}. \tag{43}$$

where e is the base of natural logarithms.

The quantity I differs from the electromagnetic mass multiplied by γc^2 only in the sign of the logarithm. If the current is close to the limiting value, i.e., if it corresponds to motion of the particles with $v_\varphi\sim c\left(J\gamma\sim\frac{eN_c}{2\pi a}\right)$, the radiation intensity of the current differs only slightly (logarithmically) from the radiation intensity of a charge. At exit from the medium the work is determined from Eqs. (29), (42), and (43). The total work for a thick plate is 2I. If, however, $\lambda_0 \gg a$, then the mass renormalization is

$$\Delta\mathcal{E}=\frac{J^2\gamma^3 2\pi^2a^4}{c^2}\int_0^\infty k^3\left(\frac{1}{\sqrt{k^2+\frac{\omega_0^2}{c^2}}}-\frac{1}{k}\right)dk$$

$$=\mu^2\frac{\omega_0^2}{c^2}\gamma\left(-k_{max}+\frac{4}{3}\frac{\omega_0}{c}\right). \tag{44}$$

Whence the radiation intensity of the magnetic moment

$$I=-\Delta\mathcal{E}-W=\frac{4}{15}\frac{\omega_0^3}{c^3}\mu^2\gamma. \tag{45}$$

amounts to a small fraction of the electromagnetic mass multiplied by γc^2. Finally, attention is merited by the fact that both Eqs. (43) and (28) are approximately the same as the radiation due to instantaneous retardation. To obtain the radiation produced by instantaneous retardation, the quantities $\ln(a/2e\lambda_0)$ and $\ln(a/2\lambda_0)$ must be replaced by $\ln(a/r)$.

3. Coherence Condition. Distribution of Force over Bunch Length

We now consider the role of the coherence factor s_ω. We can analyze charge bunches since the results are the same for the currents. From Eqs. (13), (21), and (27) the radiation intensity (when $|s_\omega|^2\neq 1$) is of the form

$$I=\frac{\omega_0^4Q^2}{c^4\pi c}\int\frac{k_\varphi^2|s_\omega|^2}{\left(\lambda_s^2-\frac{\omega^2}{v^2}\right)^2\left(\lambda^2-\frac{\omega^2}{v^2}\right)}d\omega dk. \tag{46}$$

The form-factor $|s_\omega|^2$ cuts off frequencies above $\omega_l=\frac{v\gamma}{l}$, where l is the length of the bunch in its own reference system. Assume that the coherence condition is only slightly violated. For a uniform distribution of particles over length $s_\omega=\frac{\omega_l}{\omega}\sin\frac{\omega}{\omega_l}$ with small disturbances of the coherence condition, s_ω is close to unity $|s_\omega|^2\simeq 1-\frac{2}{3}\frac{\omega^2}{\omega_l^2}$ and the reduction in radiation intensity ΔI, after substitution of the last expression in Eq. (46) and integration over frequency, is found to be

$$\Delta I=\frac{2}{3}Q^2\gamma l^2\int k\varphi_k^2\left[\frac{1}{\sqrt{k^2+\frac{\omega_0^2}{c^2}}}+\frac{1}{k}-4\frac{c^2}{\omega_0^2}\left(\sqrt{k^2+\frac{\omega_0^2}{c^2}}-k\right)\right]dk. \tag{47}$$

For conditions corresponding to the optimum case considered above ($k_{min} \ll \omega_0/c \ll k_{max}$)

$$\Delta I = \frac{\pi a}{3} Q^2 \frac{l^2}{\lambda_0^2}\gamma = \frac{e^2N^2}{12\pi a}\gamma\frac{l^2}{\lambda_0^2}. \qquad (48)$$

From a comparison with Eq. (28), we find that (48) is small if

$$l^2 \ll 6\lambda_0^2 \ln\frac{a}{2\lambda_0}. \qquad (49)$$

A more detailed calculation shows that up to $l \sim \lambda_0$ there is no appreciable reduction in the coherence effect for a wide class of particles length distributions so long as the particle density falls off appreciably at distances of order l.

Equation (49) is the coherence condition and has a simple physical meaning. In any case, the coherent radiation is that for which the dimensions of the bunch are smaller than the shortest radiated wavelength. As is well known, for transition radiation the spectral density is a weak logarithmic function of energy and extends up to frequencies of order $\omega_0\gamma$, corresponding to the shortest wavelength λ_0/γ. However, because of the Lorentz contraction the bunch dimension is l/γ. As a result, the coherence condition (49) is independent of energy γ, a result which should be emphasized again.

If the coherence condition is not satisfied but the inequality which is the inverse of (49) holds, we may assume that the denominator in Eq. (46) is independent of frequency

$$I \simeq \frac{4\pi a Q^2\gamma}{l\lambda_0^4}\int_{k_{min}}^{k_{max}} \frac{dk}{k^2\left(k^2+\frac{\omega_0^2}{c^2}\right)^2} \simeq \frac{4\pi a^2 Q^2\gamma}{l}. \qquad (50)$$

The last relation (50) is written for the condition $k_{min} \ll \omega_0/c \ll k_{max}$. We consider the distribution of forces over the length of the bunch in the reference system fixed in the bunch. When all the particles act to produce the current, the force density is given by

$$f_{zs} = F_z\rho_s(r, z);$$

$$\sigma_s(r, z) = \frac{\rho_s(r, z)}{Q} = \frac{j_{\varphi,s}(r, z)}{J},$$

that is

$$F_z = QE_{zs} - \frac{J_s}{c}H_{rs} = QE_z - \frac{J_c\gamma}{c}(H_r + \beta E_\varphi). \qquad (51)$$

We first consider the motion of the bunch into the medium.

Substituting Eqs. (8), (32) and (33) we find ($J_c = J\gamma$) for a ring characterized by $r = a$ in the medium

$$F_z^i = \frac{2i}{\pi v}\int_{k_{min}}^{k_{max}} s_\omega d\omega dk \left\{ \frac{Q^2k^2}{(\varepsilon\lambda-\lambda_s)\varepsilon\frac{\omega}{v}}\left(\frac{\frac{\omega}{v}-\varepsilon\lambda}{\frac{\omega^2}{v^2}-\lambda_s^2} - \frac{\varepsilon}{\frac{\omega}{v}+\lambda}\right) + \frac{J_s^2}{c^2}\frac{\left(\lambda_s-\beta^2\frac{\omega}{v}\right)(\lambda+\lambda_s)}{\left(\lambda_s^2-\frac{\omega^2}{v^2}\right)\left(\lambda+\frac{\omega}{v}\right)}\right\} e^{i\left(\lambda_s-\beta^2\frac{\omega}{v}\right)z_c\gamma+i\omega t_c\gamma\left(\lambda_s-\frac{\omega}{v}\right)}. \qquad (52)$$

It will be apparent that $\omega = 0$ is not a singularity of the integrand while the points $\omega = \pm\omega_0$ ($\varepsilon = 0$) lie in the lower half plane of ω. For the coherent case $|s_\omega|^2 = 1$ we need consider only linear terms in z_c. We consider the time $t_c = 0$ when the bunch is divided in half by the boundary

$$F_z^i = F^i(0) + z_c\Lambda^i. \qquad (53)$$

The integration over frequency is taken by closing the path in the upper half plane of the complex variable ω (if the path is closed in the lower half plane, it is necessary to take account of the integral over the branch cut along the real axis); we find

$$\Lambda^i = 2Q^2\int_{k_{min}}^{k_{max}} dk\left\{\frac{k^2}{\left(k^2+\frac{\omega_0^2}{c^2}\right)\left(1+\frac{\omega_0^2}{\gamma^2v^2\varkappa^2}\right)} - \frac{2k\gamma^2(k'-\beta^2k)}{k'+k\left(1+\frac{\omega_0^2}{\gamma^2k^2v^2}\right)}\right. + \frac{2J_s^2}{c^2}\int_{k_{min}}^{k_{max}} dk\left\{\frac{2c^2}{\omega_0^2}\gamma^4(k'-k)(k'-\beta^2k)^2-\varkappa\right\};$$

$$\varkappa = \sqrt{k^2+\frac{\omega_0^2}{c^2}}; \quad k' = \sqrt{k^2+\frac{\omega_0^2}{\gamma^2c^2}}. \qquad (54)$$

In the ultrarelativistic case

$$\Lambda^e \simeq 2Q^2 \int_{k_{min}}^{k_{max}} dk \left[\frac{k^2}{\varkappa} - k\left(1+\frac{\omega_0^2}{2c^2k^2}\right)\right] + \frac{2J_e^2}{c^2}\int_{k_{min}}^{k_{max}} dk\left[k\left(1+\frac{\omega_0^2}{2c^2k^2}\right)^2 - \varkappa\right] \quad (55)$$

If $J_c \simeq Q_c$, then

$$\Lambda^e \simeq \frac{2Q^2\omega_0^2}{c^2} \times \int_{k_{min}}^{k_{max}} dk \left\{-\frac{1}{\varkappa} + \frac{1}{2k}\left(1+\frac{\omega_0^2}{2c^2k^2}\right)\right\}. \quad (56)$$

The sign of Λ^e determines whether the force acts to contract or extend the bunch. It is apparent from Eq. (55) that the charge and current act in opposite directions. The charge tends to contract the bunch ($\Lambda^e_\xi < 0$) while the current tends to extend it ($\Lambda^e_J > 0$). It follows from Eq. (56) that when $J_c \simeq Q_c$ and $\lambda_0 \sim Q$ the bunch can be contracted.

However, if account is taken of the fact that the self forces are changed for the part of the bunch which has penetrated into the medium and additional extending forces come into play (in accordance with the results of B. M. Bolotovskii), the resulting forces are found to correspond to extension. Actually, if in the expressions for the force (51) we substitute the self fields (2) and (30), for the linear term in Z in the same approximation ($J_c \simeq Q_c$) we have

$$\Lambda^e_{self} = 2Q^2 \int_{k_{min}}^{k_{max}} \frac{\omega_0^2}{c^2\varkappa}\, dk. \quad (57)$$

Combining Eqs. (56) and (57) we obtain a net extensive force. This extension is stronger than the contraction. The latter follows from the distribution of forces in the part of the bunch which has not entered the medium. Substituting the appropriate free fields in vacuum in (51) gives the following linear term in z_c:

$$A = -\frac{2Q^2}{\pi v}\int_{k_{min}}^{k_{max}} \frac{dk\, d\omega\, k^2\left(\lambda - \beta^2\frac{\omega}{v}\right)\left(\varepsilon\frac{\omega}{v} - \lambda_e\right)}{\frac{\omega}{v}(\lambda_e - \varepsilon\lambda)\left(\frac{\omega^2}{v^2} - \lambda^2\right)} - \frac{2J_e^2}{\pi v\omega^2}\int_{k_{min}}^{k_{max}} d\omega\, dk \frac{\left(\lambda - \beta\varepsilon\frac{\omega}{v}\right)^2(\lambda + \lambda_e)}{\left(\lambda^2 - \frac{\omega^2}{v^2}\right)\left(\lambda + \frac{\omega}{v}\right)}. \quad (58)$$

In the ultrarelativistic limit with $J \simeq Q_c$ we obtain a contracting force

$$\Lambda \simeq -4Q^2 \int_{k_{min}}^{k_{max}} \frac{\omega_0^2}{c^2k}\gamma^2 dk, \quad (59)$$

which is considerably greater than the sum of (56) and (57) because of the factor γ^2. We can find Λ for all times $t_c < 0$ then (Eq. 59 corresponds to $t_c = 0$) and find that the time during which the contracting force operates is of order $t_{c0} \sim \frac{a}{v\gamma^2}$, that is to say, it contains γ^2 in the denominator.

If the bunch travels through a thick layer then the basic retarding force operates at exit; at entrance the radiating bunch is accelerated because of mass renormalization. This acceleration is terminated by the retardation upon exit so that the resulting energy loss can be greater than the electromagnetic mass of the bunch, as we have noted above.

For this reason great interest attaches to the deformation of the bunch upon exit from the medium. If the same conditions as given above are satisfied and $J \simeq Q_c$, the exit of the bunch is accompanied by compression both in the medium and in the vacuum. Substitution of the appropriate fields in (51) gives:

In the medium ($\gamma \gg 1$)

$$\Lambda^n = -2Q^2 \int_{k_{min}}^{k_{max}} \frac{\omega_0^2}{c^2} \frac{\gamma^2}{\sqrt{k^2 + \frac{\omega_0^2}{c^2}}}\, dk \quad (60)$$

there is a strong compression $\sim \gamma^2$ which acts for a small time interval $t_{comp} \sim \frac{1}{\omega_0\gamma^2}$:

2) In the vacuum ($\gamma \gg 1$)

$$\Lambda' \simeq -\frac{Q^2\omega_0^2}{2c^2}\int_{k_{min}}^{k_{max}} \frac{dk}{\sqrt{k^2 + \frac{\omega_0^2}{c^2}}} \frac{k^2 + \frac{1}{2}\frac{\omega_0^2}{c^2}}{k^2 + \frac{\omega_0^2}{c^2}} \quad (61)$$

there is a relatively small compressive force which acts on the bunch for a considerable greater time $\left(t_{comp} \sim \frac{1}{\omega_0}\right)$.

In conclusion, I wish to take this opportunity to thank V. L. Veksler for his helpful discussions of these problems; I am also indebted to V. L. Ginsburg, M. S. Rabinovich, and B. M. Bolotovskii for valuable comments on this work.

LITERATURE CITED

1. V. L. Ginzburg and I. M. Frank. ZhETF, 16, 15, 1946.
2. F. Ternovskii. ZhETF, 39, 171, 1960 (Soviet Physics—JETP, Vol. 12, p. 123).
3. G. M. Garibyan. ZhETF, 37, 527, 1959 (Soviet Physics—JETP, Vol. 10, p. 372).

4. K. A. Barsukov. ZhÉTF, 37, 1106, 1959 [Soviet Physics – JETP, Vol. 10, p. 787].
5. G. A. Askar'yan. ZhÉTF, 30, 584, 1956 [Soviet Physics – JETP, Vol. 3, p. 613].
6. V. I. Veksler. Atomnaya énergiya, 2, 427, 1957.
7. I. E. Tamm and I. M. Frank. DAN SSSR, 14, 109, 1937.
8. L. S. Bogdankevich. ZhÉTF, 36, No. 3, 1959 [Soviet Physics – JETP, Vol. 9, p. 589].
9. M. S. Rabinovich and L. Ioganson, ZhÉTF 37, 118, 1959 [Soviet Physics – JETP, Vol. 10, p. 83].

All abbreviations of periodicals in the above bibliography are letter-by-letter transliterations of the abbreviations as given in the original Russian journal. *Some or all of this periodical literature may well be available in English translation.* A complete list of the cover-to-cover English translations appears at the back of this issue.

Declassified in Part - Sanitized Copy Approved for Release 2011/12/14 : CIA-RDP80T00246A015600440001-0